Transparents du huitième cours

- **Modèle Standard**
- γ_5 **et Dim-Reg**
- **Espaces de dimension** z
- **Anomalies et formule de l'indice locale**

Modèle Standard

$$\mathcal{L}_{SM} = -\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \frac{1}{2} i g_s^2 (\bar{q}^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu$$

$$+\bar{G}^a \partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu$$

$$-\frac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H - \frac{1}{2} m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^-$$

$$-\frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2c_w^2} M \phi^0 \phi^0 - \beta_h \left(\frac{2M^2}{g^2} + \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-) \right)$$

$$+\frac{2M^4}{g^2}\alpha_h - igc_w(\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu)$$

$$+Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - igs_w(\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu)$$

$$-A_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu))$$

$$-\frac{1}{2} g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac{1}{2} g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu)$$

$$+g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w (A_\mu Z^0_\nu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu)$$

$$-2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - g\alpha_h M \left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^- \right)$$

$$-\frac{1}{8} g^2 \alpha_h \left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2 \phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2 H^2 \right)$$

$$-gMW^+_\mu W^-_\mu H - \frac{1}{2} g \frac{M}{c_w^2} Z^0_\mu Z^0_\mu H$$

$$-\frac{1}{2} ig \left(W^+_\mu (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W^-_\mu (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0) \right)$$

$$+\frac{1}{2}g\left(W_\mu^+(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W_\mu^-(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right)$$

$$+\frac{1}{2}g\frac{1}{c_w}(Z_\mu^0(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\frac{s_w^2}{c_w}MZ_\mu^0(W_\mu^+\phi^- - W_\mu^-\phi^+)$$

$$+igs_wMA_\mu(W_\mu^+\phi^- - W_\mu^-\phi^+) - ig\frac{1-2c_w^2}{2c_w}Z_\mu^0(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+)$$

$$+igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4}g^2W_\mu^+W_\mu^-\left(H^2 + (\phi^0)^2 + 2\phi^+\phi^-\right)$$

$$-\frac{1}{4}g^2\frac{1}{c_w^2}Z_\mu^0Z_\mu^0\left(H^2 + (\phi^0)^2 + 2(2s_w^2-1)^2\phi^+\phi^-\right)$$

$$-\frac{1}{2}g^2\frac{s_w^2}{c_w}Z_\mu^0\phi^0(W_\mu^+\phi^- + W_\mu^-\phi^+) - \frac{1}{2}ig^2\frac{s_w^2}{c_w}Z_\mu^0H(W_\mu^+\phi^- - W_\mu^-\phi^+)$$

$$+\frac{1}{2}g^2s_wA_\mu\phi^0(W_\mu^+\phi^- + W_\mu^-\phi^+) + \frac{1}{2}ig^2s_wA_\mu H(W_\mu^+\phi^- - W_\mu^-\phi^+)$$

$$-g^2\frac{s_w}{c_w}(2c_w^2-1)Z_\mu^0A_\mu\phi^+\phi^- - g^2s_w^2A_\mu A_\mu\phi^+\phi^-$$

$$-\bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \bar{u}_j^\lambda(\gamma\partial + m_u^\lambda)u_j^\lambda - \bar{d}_j^\lambda(\gamma\partial + m_d^\lambda)d_j^\lambda$$

$$+igs_wA_\mu\left(-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}_j^\lambda\gamma^\mu u_j^\lambda) - \frac{1}{3}(\bar{d}_j^\lambda\gamma^\mu d_j^\lambda)\right)$$

$$+\frac{ig}{4c_w}Z_\mu^0\{(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda)$$

$$+(\bar{u}_j^\lambda\gamma^\mu(\frac{4}{3}s_w^2 - 1 - \gamma^5)u_j^\lambda) + (\bar{d}_j^\lambda\gamma^\mu(1 - \frac{8}{3}s_w^2 - \gamma^5)d_j^\lambda)\}$$

$$+\frac{ig}{2\sqrt{2}}W_\mu^+\left((\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) + (\bar{u}_j^\lambda\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d_j^\kappa)\right)$$

$$+\frac{ig}{2\sqrt{2}}W_\mu^-\left((\bar{e}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}_j^\kappa C_{\lambda\kappa}^\dagger\gamma^\mu(1+\gamma^5)u_j^\lambda)\right)$$

$$+\frac{ig}{2\sqrt{2}}\frac{m_e^\lambda}{M}\left(-\phi^+(\bar{\nu}^\lambda(1-\gamma^5)e^\lambda)+\phi^-(\bar{e}^\lambda(1+\gamma^5)\nu^\lambda)\right)$$

$$-\frac{g}{2}\frac{m_e^\lambda}{M}\left(H(\bar{e}^\lambda e^\lambda)+i\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda)\right)$$

$$+\frac{ig}{2M\sqrt{2}}\phi^+\left(-m_d^\kappa(\bar{u}_j^\lambda C_{\lambda\kappa}(1-\gamma^5)d_j^\kappa)+m_u^\lambda(\bar{u}_j^\lambda C_{\lambda\kappa}(1+\gamma^5)d_j^\kappa\right)$$

$$+\frac{ig}{2M\sqrt{2}}\phi^-\left(m_d^\lambda(\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger(1+\gamma^5)u_j^\kappa)-m_u^\kappa(\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger(1-\gamma^5)u_j^\kappa\right)$$

$$-\frac{g}{2}\frac{m_u^\lambda}{M}H(\bar{u}_j^\lambda u_j^\lambda)-\frac{g}{2}\frac{m_d^\lambda}{M}H(\bar{d}_j^\lambda d_j^\lambda)+\frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar{u}_j^\lambda\gamma^5 u_j^\lambda)-\frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar{d}_j^\lambda\gamma^5 d_j^\lambda)$$

$$+\bar{X}^+(\partial^2-M^2)X^+ +\bar{X}^-(\partial^2-M^2)X^- +\bar{X}^0(\partial^2-\frac{M^2}{c_w^2})X^0$$

$$+\bar{Y}\partial^2 Y+igc_w W_\mu^+(\partial_\mu\bar{X}^0X^- -\partial_\mu\bar{X}^+X^0)+igs_w W_\mu^+(\partial_\mu\bar{Y}X^- -\partial_\mu\bar{X}^+Y)$$

$$+igc_w W_\mu^-(\partial_\mu\bar{X}^-X^0-\partial_\mu\bar{X}^0X^+)+igs_w W_\mu^-(\partial_\mu\bar{X}^-Y-\partial_\mu\bar{Y}X^+)$$

$$+igc_w Z_\mu^0(\partial_\mu\bar{X}^+X^+ -\partial_\mu\bar{X}^-X^-)+igs_w A_\mu(\partial_\mu\bar{X}^+X^+ -\partial_\mu\bar{X}^-X^-)$$

$$-\frac{1}{2}gM\left(\bar{X}^+X^+H+\bar{X}^-X^-H+\frac{1}{c_w^2}\bar{X}^0X^0H\right)$$

$$+\frac{1-2c_w^2}{2c_w}igM\left(\bar{X}^+X^0\phi^+ -\bar{X}^-X^0\phi^-\right)$$

$$+\frac{1}{2c_w}igM\left(\bar{X}^0X^-\phi^+ -\bar{X}^0X^+\phi^-\right)$$

$$+igMs_w\left(\bar{X}^0X^-\phi^+ -\bar{X}^0X^+\phi^-\right)$$

$$+\frac{1}{2}igM\left(\bar{X}^+X^+\phi^0-\bar{X}^-X^-\phi^0\right)\ .$$

Notations

Bosons de jauge : $A_\mu, W_\mu^\pm, Z_\mu^0, g_\mu^a$

Quarks : u_j^κ, d_j^κ et $q_j^\sigma =$ collectif pour quarks

Leptons : e^λ, ν^λ

Higgs : $H, \phi^0, \phi^+, \phi^-$

Fantômes : G^a, X^0, X^+, X^-, Y, jauge Feynman

Masses : $m_d^\lambda, m_u^\lambda, m_e^\lambda, m_h, M \leftarrow$ Masse du W

Constantes de couplage : $g = \sqrt{4\pi\alpha}$ (structure fine), $g_s =$ forte, $\alpha_h = \frac{m_h^2}{4M^2}$

Constante de tadpole β_h

Sinus et cosinus de l'angle faible : s_w, c_w

Cabibbo-Kobayashi-Maskawa : $C_{\lambda\kappa}$

Constantes de structure de SU$_3$: f^{abc}

Indice Local NCG (ac+hm)

$$⨍ P := \text{Res}_{z=0}\, \text{Tr}\,(P|D|^{-z})$$

Trace sur l'algèbre engendrée par $\mathcal{A}$, $[D, \mathcal{A}]$ et $|D|^z$, $z \in \mathbb{C}$.

$$\varphi_0(a) = \lim_{z \to 0} \text{Tr}(\gamma\, a\, |D|^{-z}), \quad \forall a \in \mathcal{A},$$

$$\varphi_n(a^0, \ldots, a^n) :=$$

$$\sum_k c_{n,k} ⨍ \gamma\, a^0 [D, a^1]^{(k_1)} \ldots [D, a^n]^{(k_n)}\, |D|^{-n-2|k|}$$

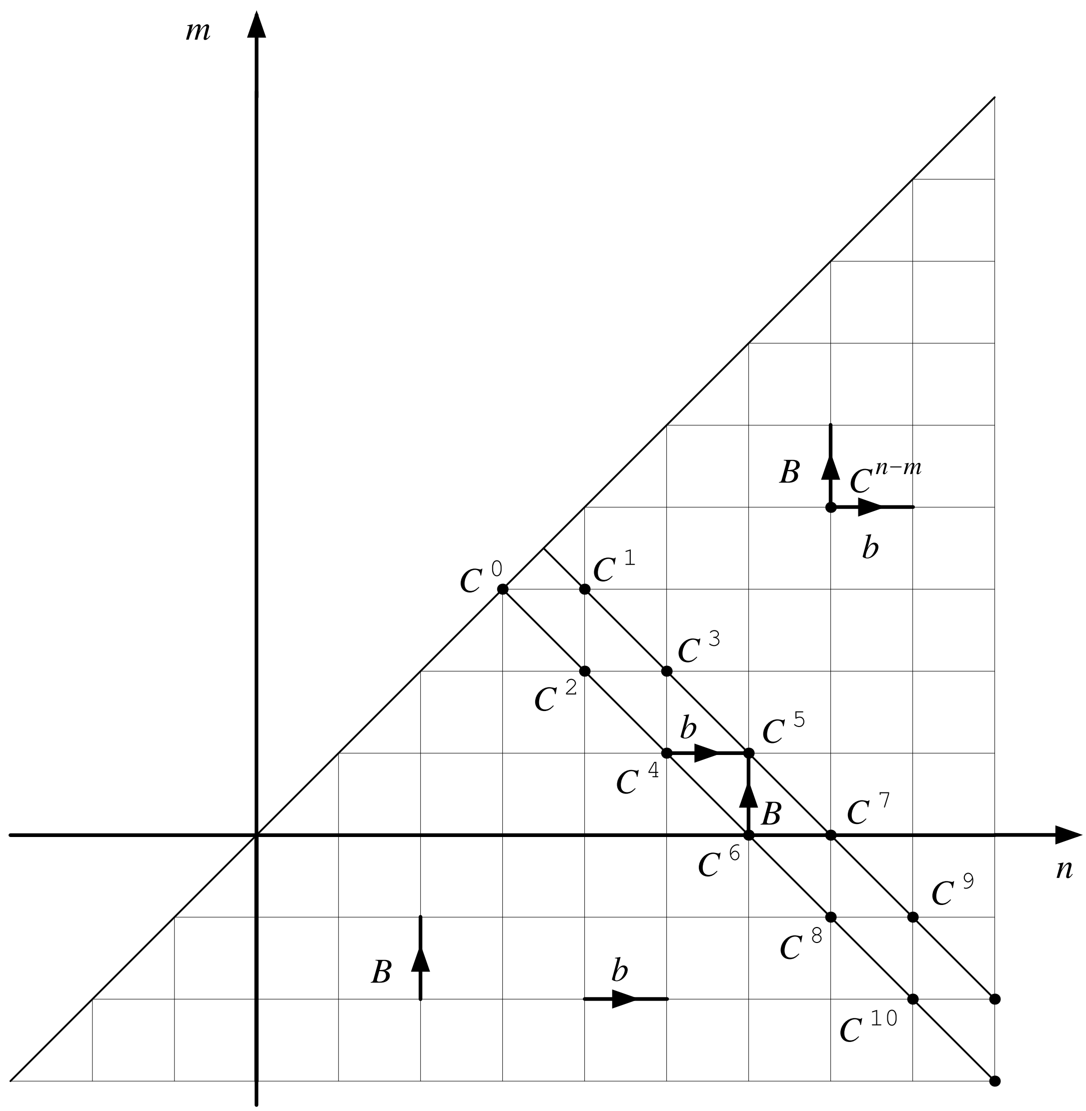
$T^{(k_i)} = \nabla^{k_i}(T)$ avec $\nabla(T) = D^2 T - T D^2$, $|k| = k_1 + \ldots + k_n$,

$$c_{n,k} = \frac{(-1)^{|k|}}{2}\, (k_1! \ldots k_n!)^{-1}$$

$$((k_1{+}1) \ldots (k_1{+}k_2{+}\ldots{+}k_n{+}n))^{-1}\, \Gamma\, (|k| + n/2)\,.$$

$(\varphi_n)_{n=0,2,\ldots}$ cocycle du (b, B)-bicomplexe de $\mathcal{A}$.

Accouplement $(\varphi_n) \in HC^*(\mathcal{A})$ avec $K_0(\mathcal{A})$ = indice D, $K_0(\mathcal{A}) \to \mathbb{Z}$.


m
n
C 0
C 1
C 2
C 3
C 4
C 5
C 6
C 7
C 8
C 9
C 10
C n−m
B
b

Dim-Reg

Espaces X_z de dimension z (ac + mm)

t'Hooft-Veltman et Breitenlohner-Maison $\Leftrightarrow$ faire le produit de l'espace-temps euclidien par un triplet spectral X_z de dimension $z \in \mathbb{C}$, Re(z) > 0

$$\mathcal{H}'' = \mathcal{H} \otimes \mathcal{H}' , \quad D'' = D \otimes 1 + \gamma_5 \otimes D_z$$

Spectre de dimensions de X_z est réduit à z.

$$\text{Trace}(e^{-\lambda D_z^2}) = \pi^{z/2} \lambda^{-z/2} , \quad \forall \lambda \in \mathbb{R}_+^*$$

Espaces X_z

$$\mathrm{Tr}_N(1_E(Z)) = \frac{1}{2}\int_E dy$$

$$D_z = \rho(z)\, F\, |Z|^{1/z}$$

$$\rho(z) = \pi^{-\frac{1}{2}}\,(\Gamma(\frac{z}{2}+1))^{\frac{1}{z}}$$

L'opérateur D_z vérifie pour $z > 0$

$$\mathrm{Tr}_N(e^{-\lambda D_z^2}) = \pi^{z/2}\,\lambda^{-z/2}\,, \quad \forall \lambda \in \mathbb{R}_+^*$$

et pour $z \neq 0$,

$$\mathrm{Tr}_N((D_z^2)^{s/2})$$

a un pole simple en $s = z$ et est absolument convergente dans $\mathrm{Re}(s/z) > 1$.

Cutoff infrarouge

$$\mathrm{Tr}'_N((D_z^2)^{-s/2}) = \frac{1}{2}\int_{|y|>1} (\rho^2\,|y|^{2/z})^{-s/2}\,dy$$

$$= \rho^{-s}\int_1^{\infty} u^{-s/z}\,du = \rho^{-s}\frac{z}{s-z}$$

$$R(\lambda, z) = \int_0^{\frac{1}{2}} (e^{-\lambda\,\rho^2\,|y|^{2/z}} - e^{-\lambda\,\rho^2\,f(|y|)^{2/z}})\,dy$$

$$|R(\lambda, z)| < C\,|\lambda|\,2^{\mathrm{Re}(-2/z)}$$

Potentiel de jauge

$(\mathcal{A},\mathcal{H},D)$ et $\mathcal{E}$ projectif de type fini sur $\mathcal{A}$,

$\mathcal{B} = \text{End}_{\mathcal{A}}(\mathcal{E})$, $\mathcal{H}' = \mathcal{E} \otimes_{\mathcal{A}} \mathcal{H}$, $D' = ?$

$$D'(\xi \otimes \eta) = \xi \otimes D\eta + \nabla(\xi)\eta$$

$$\nabla(\xi a) = (\nabla\xi)a + \xi \otimes da\,, \quad da = [D,a]$$

$$A = \Sigma a_i[D,b_i], \quad a_i, b_i \in \mathcal{A} \to \Omega^1_D \subset \mathcal{L}(\mathcal{H})$$

$$D \mapsto D + A, \quad A = A^*$$

$$u\,(D + A)\,u^* = D + \alpha_u(A)\,,$$

$$\alpha_u(A) = u[D,u^*] + u\,A\,u^*$$

Potentiel évanescent

$\mathcal{A}$ algèbre $\mathbb{Z}/2$-graduée,

$$[D, a]_- := D\,a - (-1)^{\deg(a)} a\,D$$

$\tilde{\mathcal{A}} = \{a + b\,\gamma\} \sim \mathcal{A} \oplus \mathcal{A}\,, a + b\,\gamma \mapsto (a + b,\, a - b)$

$\mathbb{Z}/2$-graduée par

$\theta \in \mathrm{Aut}(\tilde{\mathcal{A}})\,, \quad \theta(\gamma) = -\gamma\,, \quad \theta(a) = a\,, \quad \forall a \in \mathcal{A}$

$$\bar{D} = D \otimes 1\,, \quad \hat{D} = \gamma \otimes D_z\,, \quad D'' = \bar{D} + \hat{D}$$

Potentiel évanescent

$$B = [D'', \gamma]_- = 2\,\gamma\,\hat{D}$$

$$E = \frac{1}{2}\,a\,[D'', \gamma]_- = \gamma\,a\,\hat{D}$$

Intégrale fonctionnelle fermionique

Euclidien

$$\mathcal{L}_{\text{fermions}} = \langle \eta \,,\, D'' \xi \rangle$$

avec deux variables *indépendantes* ξ et η.

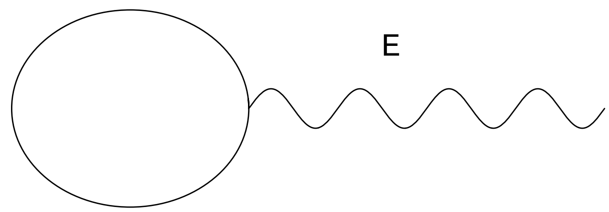
Transformations de jauge graduées

$$\alpha_u(\xi) = u\,\xi\,, \quad \alpha_u(\eta) = \theta(u)\,\eta$$

Symétrie chirale $\quad u = e^{ia\gamma}$

$$\delta\,\mathcal{L}_{\text{fermions}} = \langle \eta, \,(i\,[D,a]\,\gamma + 2\,i\,a\,\gamma\,\widehat{D})\,\xi \rangle$$

Tadpole



$$E = \gamma a \widehat{D}$$

Soit φ_0 la composante de dimension 0 du cocycle local, alors (quand $z \to 0$)

$$\mathrm{Tr}\,(E\,D''^{-1}) = -\,\varphi_0(a)$$

$$D''^{-1} = D'' \, D''^{-2} = (\bar{D} + \widehat{D}) \, D''^{-2}$$

$$D''^{2} = \bar{D}^2 + \widehat{D}^2$$

$$D''^{-2} = \int_0^\infty e^{-t\bar{D}^2} \, e^{-t\widehat{D}^2} \, dt$$

$$\mathrm{Tr}(E \, D''^{-1}) = \mathrm{Tr}(\gamma \, a\widehat{D} \, (\bar{D} + \widehat{D}) \, D''^{-2}) =$$

$$\mathrm{Tr}(\gamma \, a\widehat{D}^2 \, D''^{-2}) =$$

$$\int_0^\infty \mathrm{Tr}(\gamma \, a\widehat{D}^2 \, e^{-t\bar{D}^2} \, e^{-t\widehat{D}^2}) \, dt =$$

$$\int_0^\infty \mathrm{Tr}(\gamma \, a \, e^{-tD^2}) \mathrm{Tr}(D_z^2 \, e^{-tD_z^2}) \, dt$$

$$\mathrm{Tr}_N(D_z^2 \, e^{-tD_z^2}) = \frac{z}{2} \pi^{z/2} \, t^{-z/2-1} \quad \forall t \in \mathbb{R}_+^*$$

$$\int_0^\infty e^{-tD^2} \, t^{-z/2-1} \, dt = \Gamma(-z/2) \, |D|^z$$

Opérateurs différentiels : $OP(\mathcal{A}, \mathcal{H}, D)$

$$D^{-2}\, T \sim \sum_{0}^{\infty} (-1)^k\, \nabla^k(T)\, D^{-2k-2}$$

Lemme

Soit $P \in OP(\mathcal{A}, \mathcal{H}, D)$. Pour $n > k > 0$ et $z \to 0$,

$$\mathrm{Tr}\,(\gamma\, \hat{D}^{2k}\, (P \otimes 1)\, D''^{-2n}) =$$

$$-\frac{1}{2}\, B(k, n-k) \int\!\!\!\!\!- \gamma\, P\, D^{-2(n-k)}$$

$$\mathrm{Tr}\,(\gamma\,\widehat{D}^{2k}\,(P\otimes 1)\,D''^{-2n}) =$$

$$\frac{1}{\Gamma(n)}\int_0^\infty \mathrm{Tr}(\gamma\,\widehat{D}^{2k}\,(P\otimes 1)\,e^{-t\bar{D}^2}\,e^{-t\widehat{D}^2})\,t^{n-1}\,dt$$

$$\mathrm{Tr}_N(D_z^{2k}\,e^{-tD_z^2}) = \frac{\prod_0^{k-1}(z+2j)}{2^k}\,\pi^{z/2}\,t^{-z/2-k}$$

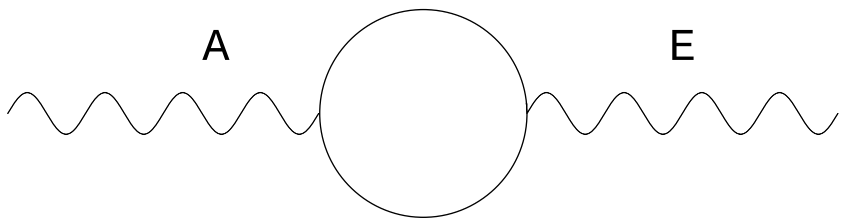
$$\int_0^\infty e^{-tD^2}\,t^{n-1-z/2-k}\,dt = \Gamma(n-z/2-k)\,|D|^{z-2(n-k)}$$

$$⨍ Q := \mathrm{Res}_{z=0}\,\mathrm{Tr}\,(Q|D|^{-z})$$

$$Q = \gamma\,P\,D^{-2(n-k)}$$

$$B(p,q) = \frac{\Gamma(p)\,\Gamma(q)}{\Gamma(p+q)}$$

Self-énergie



$$\mathrm{Tr}(E\,D''^{-1}\,A\,D''^{-1}) =$$

$$\sum_0^\infty (-1)^{n+1} \frac{1}{2n+2} ⨍ \gamma\, a\, \nabla^n(B)\, D^{-2n-2}$$

où $B = D\,A + A\,D = dA + A'$ avec

$$dA = \sum\,[D, a_i]\,[D,\, b_i]\,, \qquad A' = \sum\, a_i\, \nabla(b_i)$$

$$\mathrm{Tr}(E\,D''^{-1}\,A\,D''^{-1}) =$$

$$\mathrm{Tr}(\gamma\,a\widehat{D}\,(\bar{D} + \widehat{D})\,D''^{-2}\,A\,(\bar{D} + \widehat{D})\,D''^{-2}) =$$

$$\mathrm{Tr}(\gamma\,a\widehat{D}^2\,D''^{-2}\,A\,\bar{D}\,D''^{-2}) +$$

$$\mathrm{Tr}(\gamma\,a\widehat{D}\,\bar{D}\,D''^{-2}\,A\,\widehat{D}\,D''^{-2}) =$$

$$\mathrm{Trace}(\gamma\,a\widehat{D}^2\,D''^{-2}\,(A\,\bar{D} + \bar{D}\,A)\,D''^{-2})$$

$$(A\,\bar{D} + \bar{D}\,A) = B \otimes 1\,, \qquad B = dA + A'$$

$$D^{-2}\,T \sim \sum_0^\infty (-1)^k\,\nabla^k(T)\,D^{-2k-2}$$

$$\mathrm{Trace}(E\,D''^{-1}\,A\,D''^{-1}) =$$

$$\sum_0^\infty (-1)^n\,\mathrm{Tr}(\gamma\,a\widehat{D}^2\,(\nabla^n(B) \otimes 1)\,D''^{-2n-4})\,.$$

$$D^2 \, T \, D^{-2} = T + \nabla(T) \, D^{-2} = (1 + \epsilon)(T)$$

$$\Theta(T) = \sum_{1}^{\infty} \frac{(-1)^{n+1}}{n} \, \nabla^n(T) \, D^{-2n}$$

$$\pi(z) = \frac{z}{e^z - 1}, \quad \pi(-z) = e^z \, \pi(z)$$

$$\mathrm{Tr}(E \, D''^{-1} \, A \, D''^{-1}) = -\frac{1}{2} ⨍ \gamma \, a \, \pi(\Theta)(B) \, D^{-2} =$$

$$-\frac{1}{2} ⨍ \gamma \, A \, \pi(\Theta)([D, a]) \, D^{-2}$$

$$⨍ X \, \pi(\Theta)(Y) \, D^{-2} = ⨍ Y \, \pi(\Theta)(X) \, D^{-2}$$

Self-énergie en $D = 2$

Cocycle de Hochschild $\qquad \varphi \to \int_\varphi$

$$\int_\varphi a_0\, da_1 \cdots da_n = \varphi(a_0, a_1, \cdots, a_n)$$

$$\int_\varphi a\, \omega = \int_\varphi \omega\, a, \quad \forall a \in \mathcal{A}$$

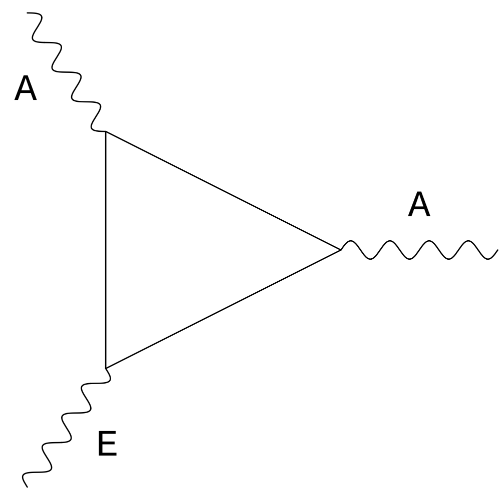
$$\varphi_2(a_0, a_1, a_2) = \frac{1}{4} ⨍ \gamma\, a_0\, [D, a_1]\, [D, a_2]\, D^{-2}$$

$$\mathrm{Tr}(E\, D''^{-1}\, A\, D''^{-1}) = -2 \int_{\varphi_2} A\, da$$

Tadpole $= 0 \Rightarrow \varphi_2$ cyclique $\Rightarrow$

$$\mathrm{Tr}(E\, D''^{-1}\, A\, D''^{-1}) = -2 \int_{\varphi_2} a\, dA$$

ABJ triangle



$$\mathrm{Tr}(E\,D''^{-1}\,A\,D''^{-1}\,A\,D''^{-1})$$

ABJ en $D = 2$

$$\mathrm{Tr}(E\, D''^{-1}\, A\, D''^{-1}\, A\, D''^{-1}) = 2 \int_{\varphi_2} a\, A^2$$

1. φ_2 est un deux cocycle cyclique* dans la classe locale.
2. Pour tout $a \in \mathcal{A}$ et A

$$\mathrm{Tr}(E\, D''^{-1}\, A\, D''^{-1}\, A\, D''^{-1}) - \mathrm{Tr}(E\, D''^{-1}\, A\, D''^{-1})$$

$$= 2 \int_{\varphi_2} a\, (dA + A^2)$$

*on suppose tadpole$= 0$

$$\mathrm{Tr}(E\,D''^{-1}\,A\,D''^{-1}\,A\,D''^{-1}) =$$

$$\mathrm{Tr}(\gamma\,a\widehat{D}\,(\bar{D} + \widehat{D})\,D''^{-2}\,A\,(\bar{D} + \widehat{D})\,D''^{-2}\,A\,(\bar{D} + \widehat{D})\,D''^{-2})$$

Nombre impair de $\widehat{D}$ donne 0

Termes avec 4 fois $\widehat{D}$,

$$T_4 = \mathrm{Tr}(\gamma\,a\widehat{D}^2\,D''^{-2}\,A\,\widehat{D}\,D''^{-2}\,A\,\widehat{D}\,D''^{-2}) =$$

$$-\mathrm{Tr}(\gamma\,a\widehat{D}^4\,D''^{-2}\,A\,D''^{-2}\,A\,D''^{-2})$$

car $\widehat{D} = \gamma \otimes D_z$ anticommute avec A (à cause de γ)

Termes en $\hat{D}^2$

$$T_1 = \mathrm{Tr}(\gamma\, a\hat{D}^2\, D''^{-2}\, A\, \bar{D}\, D''^{-2}\, A\, \bar{D}\, D''^{-2})$$

$$T_2 = \mathrm{Tr}(\gamma\, a\hat{D}\, \bar{D}\, D''^{-2}\, A\, \hat{D}\, D''^{-2}\, A\, \bar{D}\, D''^{-2}) =$$

$$\mathrm{Tr}(\gamma\, a\hat{D}^2\, \bar{D}\, D''^{-2}\, A\, D''^{-2}\, A\, \bar{D}\, D''^{-2})$$

$$T_3 = \mathrm{Tr}(\gamma\, a\hat{D}\, \bar{D}\, D''^{-2}\, A\, \bar{D}\, D''^{-2}\, A\, \hat{D}\, D''^{-2}) =$$

$$\mathrm{Tr}(\gamma\, a\hat{D}^2\, \bar{D}\, D''^{-2}\, A\, \bar{D}\, D''^{-2}\, A\, D''^{-2})$$

Lemme

$$\mathrm{Tr}(\widehat{D}^{2k}\, P_0\, D''^{-2}\, P_1\, D''^{-2}\, P_2\, D''^{-2}) =$$

$$\sum c(a,b,k) ⨍ P_0 \nabla^a(P_1)\, \nabla^b(P_2)\, D''^{-2(a+b+3-k)}$$

$$c(a,b,k) = (-1)^{a+b+1}\,\frac{1}{2}\,\frac{((k-1)!(a+b+2-k)!}{b!\,(a+1)!(a+b+2)}$$

$$D''^{-2}\, P_1\, D''^{-2}\, P_2 \sim$$

$$\sum d(a,b)\, \nabla^a(P_1)\, \nabla^b(P_2)\, D''^{-2(a+b+2)}$$

avec

$$d(a,b) = (-1)^{a+b} \sum_{0\leq c\leq b} \frac{(a+c)!}{a!\, c!} =$$

$$(-1)^{a+b}\, \frac{(a+b+1)!}{b!\,(a+1)!}$$

$$\frac{1}{2}\, \frac{(a+b+1)!}{b!\,(a+1)!}\, \frac{(k-1)!(a+b+2-k)!}{(a+b+2)!} =$$

$$\frac{1}{2}\, \frac{((k-1)!(a+b+2-k)!}{b!\,(a+1)!(a+b+2)}$$

$$T_4 = -\mathrm{Tr}(\gamma\, a \widehat{D}^4\, D''^{-2}\, A\, D''^{-2}\, A\, D''^{-2})$$

$$= \frac{1}{4} ⨍ \gamma\, a\, A^2\, D^{-2}$$

$$T_2 = \mathrm{Tr}(\gamma\, a \widehat{D}^2\, \bar{D}\, D''^{-2}\, A\, D''^{-2}\, A\, \bar{D}\, D''^{-2}$$

$$= -\frac{1}{4} ⨍ \gamma\, a\, D\, A^2\, D^{-3} = \frac{1}{4} ⨍ \gamma\, a\, A^2\, D^{-2}$$

$$\Downarrow$$

$$T_2 + T_4 = 2 \int_{\varphi_2} a\, A^2$$

$$T_1 = \mathrm{Tr}(\gamma\, a \widehat{D}^2\, D''^{-2}\, A\, \bar{D}\, D''^{-2}\, A\, \bar{D}\, D''^{-2})$$

$$= -\frac{1}{4} ⨍ \gamma\, a\, A\, D\, A\, D^{-3}$$

$$T_3 = \mathrm{Tr}(\gamma\, a \widehat{D}^2\, \bar{D}\, D''^{-2}\, A\, \bar{D}\, D''^{-2}\, A\, D''^{-2})$$

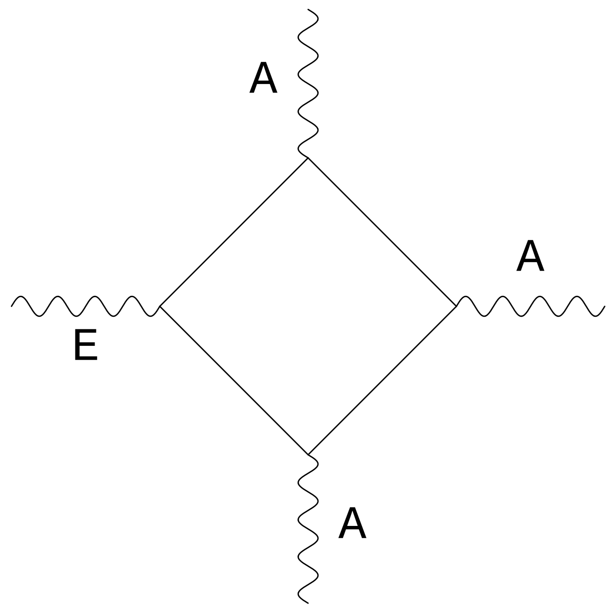
$$-\frac{1}{4} ⨍ \gamma\, a\, D\, A\, D\, A\, D^{-4} = -\frac{1}{4} ⨍ \gamma\, D\, a\, A\, D\, A\, D^{-4}$$

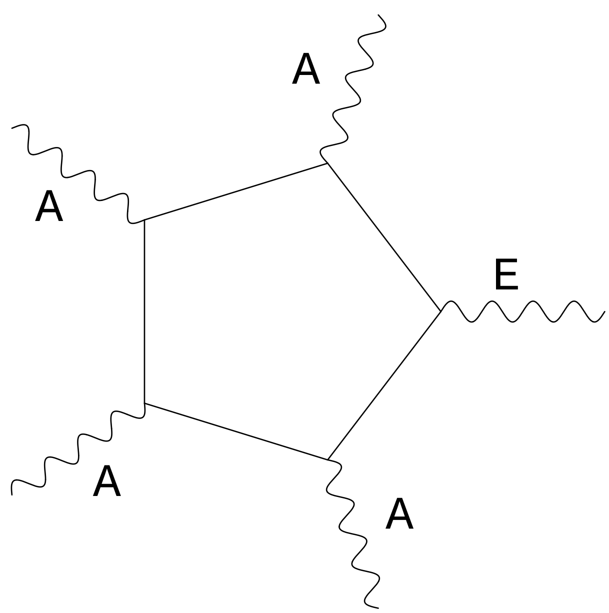
$$= \frac{1}{4} ⨍ \gamma\, a\, A\, D\, A\, D^{-3} = -T_1$$

$$\Downarrow$$

$$T_1 + T_3 = 0$$

$$D = 4$$





Fermions $\longrightarrow$ Géométrie

Fermions	$\psi \in \mathcal{H}$
Symétries internes	**Int**$(\mathcal{A})$ $f \longrightarrow u\,f\,u^*$
Bosons de Jauge	**Fluctuations internes**

Action Bosonique = Action Spectrale

(ac+ac)

$N(\Lambda) = \#$ valeurs propres de D dans $[-\Lambda, \Lambda]$.

$$N(\Lambda) = \langle N(\Lambda) \rangle + N_{\text{osc}}(\Lambda)$$

$$\langle N(\Lambda) \rangle = S_\Lambda(D) = \sum_{k \in S} \frac{\Lambda^k}{k} ⨍ |ds|^k + \zeta_D(0)\,,$$

$$\zeta_D(s) = \text{Trace}(|D|^{-s})$$

Modèle Standard en couplage minimal

$$\mathcal{L}_E + \mathcal{L}_G + \mathcal{L}_{GH} + \mathcal{L}_H + \mathcal{L}_{Gf} + \mathcal{L}_{Hf}$$

Action Spectrale (ac+ac)

$$S = \int d^4x \sqrt{g}\ (1/2\kappa_0^2\, R - \mu_0^2(H^* H)$$

$$+ a_0\, C_{\mu\nu\rho\sigma}\, C^{\mu\nu\rho\sigma} + \ b_0\, R^2 + c_0\, {}^*R^*R + d_0\, R;_\mu{}^\mu$$

$$+\ e_0 + 1/4\, G^i_{\mu\nu}\, G^{\mu\nu i} + 1/4\, F^\alpha_{\mu\nu}\, F^{\mu\nu\alpha}$$

$$+ 1/4\, B_{\mu\nu}\, B^{\mu\nu} + |D_\mu\, H|^2 - \xi_0\, R|H|^2 + \lambda_0 (H^* H)^2)$$

Modèle Standard

$$X = M \times F$$

$$\mathcal{A} = \mathcal{A}_M \otimes \mathcal{A}_F \ , \ \mathcal{H} = \mathcal{H}_M \otimes \mathcal{H}_F \ ,$$

$$D = D_M \otimes 1 + \gamma_5 \otimes D_F$$

$$\mathcal{A}_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$$

$$\mathcal{H}_F = Q \oplus L \oplus \bar{Q} \oplus \bar{L}$$

$$Q = \begin{pmatrix} u_L & u_R \\ d_L & d_R \end{pmatrix} , \quad L = \begin{pmatrix} \nu_L & ? \\ e_L & e_R \end{pmatrix}$$

Action de $\mathcal{A}$ sur $\mathcal{H}_F$

$$a = (\lambda, q, m) \in \mathcal{A}\,, \quad q = \begin{pmatrix} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{pmatrix}$$

$$a\,u_R = \lambda\,u_R \qquad a\,u_L = \alpha\,u_L - \overline{\beta}\,d_L$$

$$a\,d_R = \overline{\lambda}\,d_R \qquad a\,d_L = \beta\,u_L + \overline{\alpha}\,d_L\,.$$

$a\,\overline{f} = \lambda\,\overline{f}$ si f est un lepton

$a\,\overline{f} = m\,\overline{f}$ si f est un quark

$$\gamma(f_R) = f_R\ ,\ \gamma(f_L) = -f_L$$

Espace Fini

$$D_F = \begin{pmatrix} Y & 0 \\ 0 & \bar{Y} \end{pmatrix}$$

$$Y = Y_q \otimes 1_3 \oplus Y_\ell$$

$$Y_q = \begin{pmatrix} 0 & 0 & M_u & 0 \\ 0 & 0 & 0 & M_d \\ M_u^* & 0 & 0 & 0 \\ 0 & M_d^* & 0 & 0 \end{pmatrix}$$

$$Y_\ell = \begin{pmatrix} 0 & 0 & M_e \\ 0 & 0 & 0 \\ M_e^* & 0 & 0 \end{pmatrix}$$

Fluctuations internes → Bosons

$$A = \sum a_i[D, a_i'] \quad a_i, a_i' \in \mathcal{A}$$

$$\sum a_i[\gamma_5 \otimes D_F, a_i'] \ \rightarrow \ \textbf{Higgs}$$

$$\begin{pmatrix} 0 & X \\ X' & 0 \end{pmatrix}$$

$$X = \begin{pmatrix} M_u\,\varphi_1 & M_u\,\varphi_2 \\ -M_d\,\overline{\varphi}_2 & M_d\,\overline{\varphi}_1 \end{pmatrix}$$

$$X' = \begin{pmatrix} M_u^*\,\varphi_1' & M_d^*\,\varphi_2' \\ -M_u^*\,\overline{\varphi}_2' & M_d^*\,\overline{\varphi}_1' \end{pmatrix}$$

$\sum a_i[D_M \otimes 1, a_i']$ $\rightarrow$ **potentiels de jauge**

$a_i = (\lambda_i, q_i, m_i)$, $a_i' = (\lambda_i', q_i', m_i')$

$U(1)$ potentiel de jauge $\Lambda = \Sigma\, \lambda_i\, d\, \lambda_i'$

$SU(2)$ potentiel de jauge $Q = \Sigma\, q_i\, d\, q_i'$

$U(3)$ potentiel de jauge $V = \Sigma\, m_i\, d\, m_i'$.

Hypercharges

$$D \mapsto \tilde{D} = D + A + JAJ^{-1} \qquad A = A^*$$

$$\text{trace } V = \Lambda$$

$$V = V' + \frac{1}{3}\,\Lambda$$

V' est un potentiel de jauge $SU(3)$

$$\begin{pmatrix} \frac{4}{3}\Lambda + V' & 0 & 0 & 0 \\ 0 & -\frac{2}{3}\Lambda + V' & 0 & 0 \\ 0 & 0 & Q_{11} + \frac{1}{3}\Lambda + V' & Q_{12} \\ 0 & 0 & Q_{21} & Q_{22} + \frac{1}{3}\Lambda + V' \end{pmatrix}$$

$$\begin{pmatrix} -2\Lambda & 0 & 0 \\ 0 & Q_{11} - \Lambda & Q_{12} \\ 0 & Q_{21} & Q_{22} - \Lambda \end{pmatrix}$$

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